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THE CRITERION OF EQUAL INTEGRATED QUALITY:

An improved method of planning DBS systems

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Summary

A new technique for use in international planning of satellite broadcasting is proposed. The planning procedure would use this technique to provide an equal weighted mean quality for all circuits, as opposed to conventional techniques which would equalise the minimum quality (usually that exceeded for all but a given small percentage of the time).

As a consequence there is a different trade-off between the necessary transmit power on board the satellite and the outage time. This compromise leads to more equitable access to spectrum especially for areas where rain attenuation is high.

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1. INTRODUCTION

The CCIR is looking at high definition television (HDTV)[1] and methods of broadcasting it[2].

In an earlier Report[3] we showed that it is possible to identify ways in which all countries could broadcast HDTV even at frequencies as high as 22.5 to 23 GHz. Although rain attenuation can be high, most countries can achieve an acceptable service with realistic transmitter power on the satellite.

However difficulties have been encountered when the conventional planning techniques are used at higher frequencies. Planning could be based on ensuring that good reception is achievable in 'clear sky' conditions (i.e. when there is no rain). This approach leads to unacceptable results when rain attenuation is included: the time for which the signal is significantly degraded is excessive in some tropical areas. In other words, too low a transmit power has been chosen for the satellite. An alternative would be to ensure that, for all but a given minimum time, (say 0.1% of the year), a minimum quality level is achieved. This approach leads to excessively high transmit powers on the satellite unless a 'rain cap' - a maximum permitted allowance for rain attenuation - is applied. The compromise used at the World Administrative Radio Conference on satellite broadcasting (WARC 77)[4] was to set a target quality which had to be exceeded for all but 1% of the worst month. This works reasonably well at 12 GHz where rain attenuation is not a major problem. At higher frequencies this is a less useful compromise as it still leads to high transmit power requirements (albeit somewhat lower than those found when the 0.1% approach is used).

The high transmit powers which are necessary using the old planning methods result in some countries not achieving equitable access to the frequency band under consideration. As a consequence there is often reluctance to consider the higher frequencies for satellite broadcasting.

This Report proposes an improved international planning method using the technique of equal integrated quality. It shows that the simple planning methods can be extended to give a new, more equitable planning method. The Report shows that this method gives similar results to the method used at WARC 77, especially when rain attenuation is low. However when the range of rain attenuation increases, instead of ruling out satellite broadcasting by insisting on excessively high transmit powers to serve countries with high rainfall, it produces a more equitable solution for all countries. There is a trade-off between the quality of service provided for the majority of the time on the one hand and the time for which the service is unacceptable on the other. With the new technique a better balance is achieved, which in turn leads to less demanding satellite power requirements for many countries and hence more efficient spectrum utilisation will be possible.

2. THE EXISTING PLANNING METHODS

2.1 General objectives

When planning a broadcasting satellite service two quality factors are considered desirable:

(i) good quality is available for the majority of the time

(ii) outage times are severely limited.

If these two constraints are met, then a satisfactory service results.

An example of this can be seen in the objectives used for planning the broadcasting satellite service[5]:

> "The quality of a picture on the screen of a television receiver in the broadcasting-satellite service depends on the signal-to-noise ratio, distortions in the transmission chain and the level and type of possible interference.
>
> The desired quality values (e.g. signal-to-noise ratio) should be maintained for 99% of the time. Two further conditions must be added to the quality factor:
>
> - uninterrupted service for 99.9% of the time (with frequency modulation, this means that the carrier-to-noise ratio must remain above the threshold for 99.9% of the time).

- the elimination of the truncation noise phenomenon, which is inherent in the frequency modulation technique."*

2.2 Existing techniques for meeting the different objectives

There are three conventional criteria which can be proposed for service objectives. They are to give

1. optimum clear sky quality
2. limited outage time

and 3. defined quality exceeded for n% of the worst month

These three methods are all based on anchoring the curve of quality versus percentage time at a fixed point. The aim is that by anchoring the curve at a suitably chosen point then the general objectives stated in Section 2.1 are met (the general objectives are in fact methods 1 and 2 above). However, when rain attenuation plays a significant part, both general objectives cannot always be simultaneously satisfied unless an excessively high power is used on board the satellite.

Fig. 1† shows some of the problems of using the clear sky quality as the method of planning. The example shows how, at 20 GHz, the outage times may be acceptable if rainfall is low, but not necessarily acceptable if rain attenuation is high.

Fig. 2 shows how, on the contrary, limiting the outage times leads to excessively high transmit powers for countries in which the rain attenuation is large (the need for a high carrier-to-noise ratio, as indicated in the figure, leads to a high transmit power).

It would appear that a compromise between these two extremes may be more suitable. Obviously at some frequencies the rain attenuation will be too high for satellite broadcasting as currently proposed. The upper frequency will be different for different

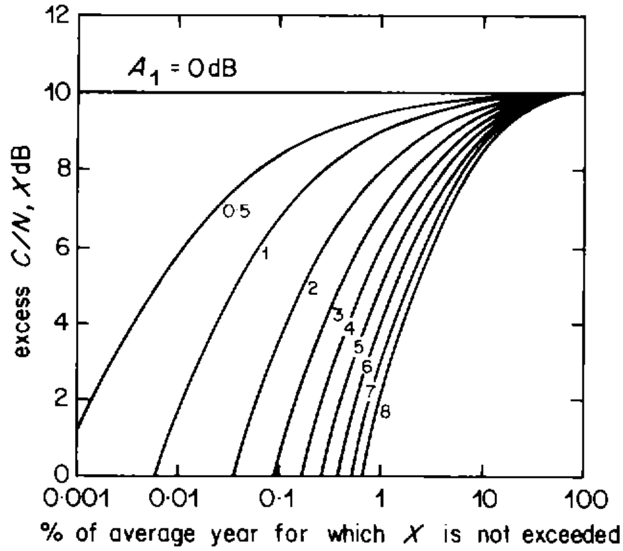

Fig. 1 - The effect of planning on the basis of equal clear-sky quality for all countries.

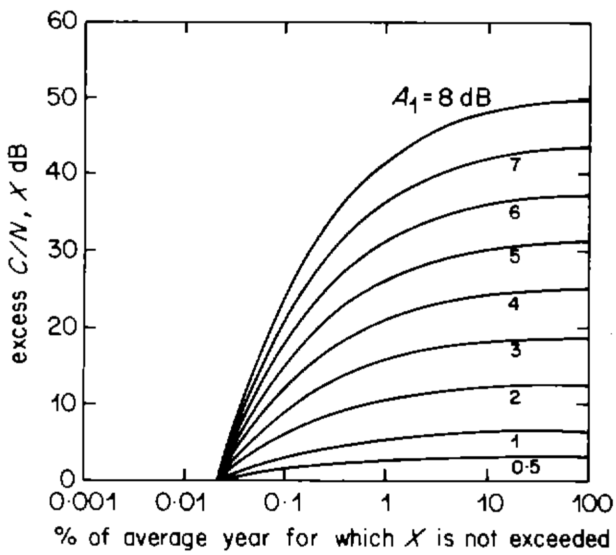

Fig. 2 - The effect of planning on the basis of equal outage time for all countries.

countries. It should be stressed that there is a great uncertainty about the rain parameters and consequent attenuations for very small percentages of the time in many areas, especially those suffering high rainfall[6,7]. Thus it is dangerous to put undue emphasis on any particular performance indicator. It is clear that most attention must be paid to achieving acceptable signal quality reasonably economically for the majority of the time. Nevertheless more attention must be paid to the signal at times near threshold than for the *same* time interval in clear sky conditions.

The use of a planning method giving a defined quality for n% worst-month is a compromise. It was used at WARC 77 for planning the 12 GHz band. The levels of rain attenuation at 12 GHz are small and

* Although this extract refers to percentages of *time*, it should be noted that WARC 77[4] used percentage of the *worst month* in its service and quality objectives.

† In these examples we have attempted to provide information in a form suitable for use in any country. This is difficult because of the range of rain zones and elevation angles found in practice. Thus, in these curves we have used the parameter 'A_1': the attenuation exceeded for 1% of the time. This parameter is discussed in Appendix 2. To give some idea of representative values at 20 GHz, for London A_1 is 1.2 dB, for Barcelona A_1 is 2.2 dB; for Kinshasa A_1 is 3.5 dB and for the parts of Africa suffering the highest rain rates A_1 is 6 to 6.5 dB; assuming in each case that the satellite is situated at a relative longitude of 15°, and using the current CCIR rain model[6].

The 'excess C/N', X, is the amount by which the C/N exceeds the threshold value, and is used here as an indication of quality (see Section 3).

as a consequence the defined minimum level of quality or a little higher will be achievable for most of the time. The world-wide rain statistics are not known sufficiently accurately to say exactly for what percentage of time a service will be below this threshold minimum quality.

However as FM degrades reasonably gracefully, any small reduction in received carrier-to-noise ratio can be tolerated for a short time. Thus the modulation system allows a little flexibility in planning. As a consequence the method is an acceptable compromise at 12 GHz.

At higher frequencies, the *n*%-worst-month method becomes less suitable. Fig. 3 shows an example of a simple planning exercise. Some countries suffer unacceptable outage times, whilst others may have suboptimum clear-sky quality.

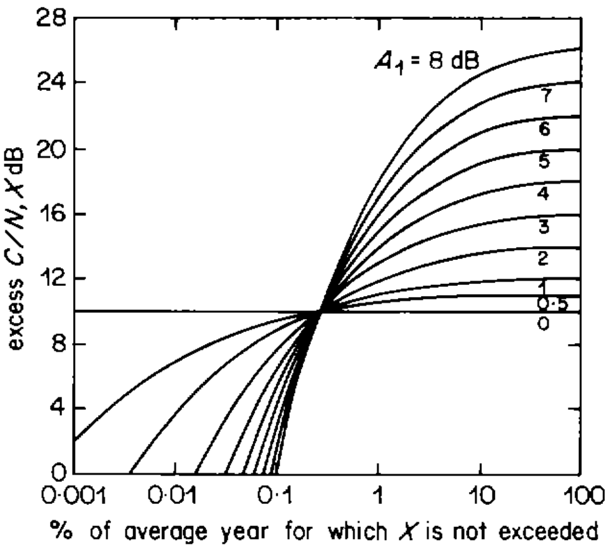

Fig. 3 - The effect of planning on the basis of equal n%-worst-month quality for all countries

Choosing an appropriate quality level and value for *n* is difficult. 1% was chosen at WARC 77 but only just meets the requirements even with the restricted range of rain attenuation assumed at 12 GHz. At WARC 77 the problems of high transmit powers were recognised and a limit imposed on the extra e.i.r.p. allocated to compensate for rain attenuation: the so-called 'rain cap'. (2 dB was used).

Thus a technique similar to defining the quality for *n*% of the worst month is needed, but with the flexibility to cope with the range of rain attenuation found in practice. In particular it is desirable to adopt a requirement permitting reasonable transmit powers.

The concept of an 'equal-integrated-quality' criterion is proposed to meet this need.

3. THE PROPOSED EQUAL-INTEGRATED-QUALITY CRITERION

3.1 Development of the idea

In Section 2 we discussed the desirable features of a planning method and the way in which the existing methods are unsatisfactory.

What we wish to achieve is a compromise; the signal quality should be good for most of the time, without 'over-engineering' the system, to balance the subjective effects of low signal strength for the short periods when rain attenuation is high. To look at this another way, we are seeking a technique which ensures that the overall quality of service is acceptable *on average*, for all countries, irrespective of the different rain characteristics they endure.

Let us take a simple example of averaging and develop it. Fig. 4 shows the expected variation of quality with the time for which it will not be exceeded, for two hypothetical places. The proposal is to average the quality so that the two means are equal. If the averaging is applied over the same interval, this is equivalent to equalising the areas under the two curves; hence the idea of equal integrated quality. The e.i.r.ps (effective isotropic radiated powers) of the two satellites are adjusted to achieve this.

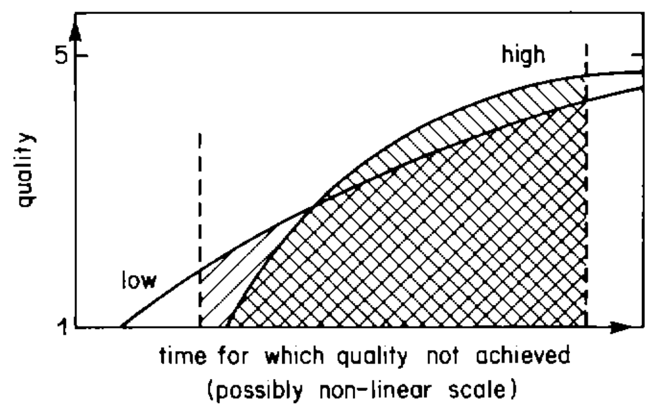

Fig. 4 - The concept of equal integrated quality.

Area ▨ *equals area* ▧

When developing the idea of an average quality it is desirable to bear in mind the fact that the viewing population is, of course, less tolerant of poor quality signals than it is of good ones. In general, viewers expect good quality and only notice the signal quality when it degrades. Thus some form of weighted mean is appropriate, to emphasise the effect of low signal quality and thus to minimise the time for which it occurs.

Although a linear function of time could be used, this places too much emphasis on clear sky conditions. A useful solution is to use a logarithmic scale for the time axis. To do this implies that the integration must start at some defined lower limit of time. However, this restriction is in any case desirable since it is reasonable to neglect outages or poor performance which only amount in total to a very small proportion of the year. It is shown in Appendix 4 that this technique of integrating over a range of log (time) is equivalent to calculating a weighted mean value of quality. In the work that follows, the *percentage* time will be used, to allow direct use of the CCIR propagation statistics which are based on percentage of the year.

It is desirable to define an objective measure of quality before progressing formally to the definition of equal integrated quality. Because the absolute subjective quality depends upon the specific transmission system used, it is more useful to use the received carrier-to-noise ratio (C/N) as the objective measure of quality in planning, as, in most cases, the subjective quality relates closely to the carrier-to-noise ratio. Sometimes the C/N falls so low that the quality of reception is unacceptable. The proposal is thus to identify a threshold C/N. Below this level the perceived quality is considered to be uniformly unsatisfactory: the signal makes no contribution to the perceived quality, nor should it, by definition, to the integrated quality. Contribution to the integrated quality can only occur when the C/N is above the threshold value.

This now gives enough information to define the integrated quality.

3.2 Definition

The integrated quality measure is defined as

$$I_{t_1} = \int_{t=t_1}^{t=t_2} X \; d(\log_{10} t), \qquad (3.1)$$

where I_{t_1} is the integrated quality measure with the lower limit set at t_1;

X is the excess carrier-to-noise ratio (measured in dB),

i.e.
$$X = (C/N - C/N_{\text{threshold}}), \quad \text{if } (.) \geqslant 0 \qquad (3.2)$$
$$= 0 \quad \text{if } (.) < 0;$$

t = percentage time for which X is not exceeded;

t_1 = percentage time corresponding to the lower limit of integration;

t_2 = percentage time corresponding to the upper limit, i.e. 100%.

In planning, the e.i.r.p. of each satellite would be adjusted to render the value of I_{t_1} the same for all services. Section 4 gives details of how this would be achieved in practice, as well as expanding on the properties of the integrated quality measure.

It is worth recognizing here that, for a hypothetical country having no rain attenuation, the C/N achieved must provide good quality and is the target C/N for the system.

3.3 The benefits from using equal integrated quality

As noted earlier, the integrated quality approach provides an equitable balance in the planning of satellite broadcasting. The best way of explaining the merits of the approach is to consider an example.

Let us consider the problem of providing a service worldwide, at 20 GHz. This was used as an example in Section 2.2 and so we can draw from earlier results. Section 2.2 showed that the existing planning methods had not got the flexibility to cope with the wide range of rain attenuation found in practice. The compromise of providing all countries with a similar service for 1% of the month leads to a need for uneconomically high e.i.r.ps on some satellites whilst still not avoiding periods of unacceptable quality.

Fig. 5 shows the performance of a system planned using equal integrated quality as a criterion. In this example it has been assumed that the minimum threshold for the system is 10 dB C/N (approximately corresponding to the f.m. threshold in some systems) and good performance is achieved at 20 dB C/N. This is typical of the systems currently under discussion[3]. The range over which the integration is performed uses 0.01% of the time as the lower limit. Details of the calculations to derive this curve can be found in the Appendices, with the general method discussed in Section 4.

The first thing that we notice about the curves derived using the equal-integrated-quality criterion is that they are similar to the curves derived by the 1%-worst-month technique. Indeed for areas where rain attenuation is low all the curves pass through a point. With the range of integration chosen (from 0.01%

time) Section 5 shows that the abscissa of this point is 0.27% of the year. (Note that 1%-worst-month statistics are taken to be about equivalent to those for 0.29% of the year. By slightly modifying the lower limit of integration, the integrated quality technique could be adjusted to give exactly identical results to those obtained by the 1%-worst-month planning method in areas of low rain attenuation).

The differences between the conventional planning technique and the equal-integrated-quality technique become apparent as rain attenuation increases. Normally the e.i.r.p. of the satellite would be increased to compensate for high rain attenuation. This leads to demands which are technically and economically difficult to satisfy. The result of using the equal-integrated-quality technique is to reduce the required satellite e.i.r.p. and render it unnecessary to use the crude technique of applying a rain cap to get a usable result.

As can be seen by comparing Figs. 3 and 5, the reduction in e.i.r.p. can be quite large. In the example, a rainy country would find that a 6 dB power reduction is permitted by the technique at 20 GHz. This still provides a good quality service.

The reduction in e.i.r.p. is made possible by an increase in the time that the system is operating below threshold. This increase in time is small and may be readily accepted given the alternative of having no service at all. Fig. 3 shows that the outage time may be as high as 0.1% of the year in extreme cases even with normal planning techniques. The equal-integrated-quality method only increases this to about 0.2% of the year (as shown in Fig. 5).

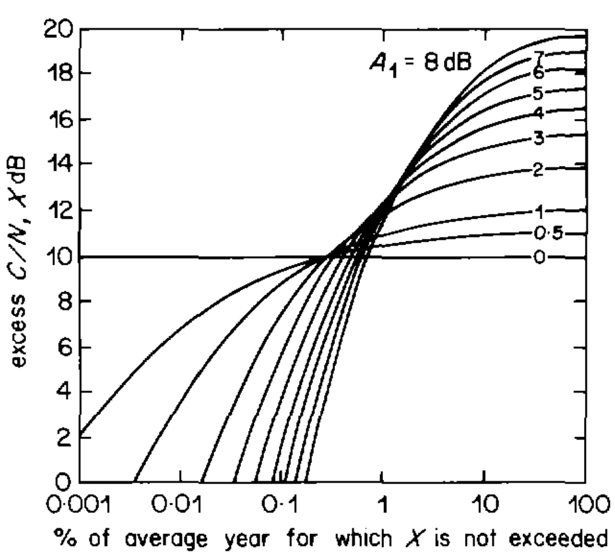

Fig. 5 - The effect of planning on the basis of equal integrated quality.

It is this favourable balance between e.i.r.p. and small adjustments in outage times which is the basis of the new proposal. The benefits of realistic access to spectrum afforded by reasonable satellite powers outweigh the small increase in outage time. (The actual value of outage time is still the subject of study. Existing propagation information in the CCIR is thought to be pessimistic in some circumstances[6, 7] and the degradation in outage time may be much smaller than currently predicted for the low-latitude, high-rain-rate countries).

4. APPLICATION OF THE IMPROVED METHOD

This Section shows how to apply the method in practice.

It is obviously necessary to develop certain mathematical tools. First, starting from the definition given in Section 3 we need to know how to evaluate the integrated quality $I_{0.01}$ for a given scenario with a particular clear-sky C/N and rain attenuation. This is derived in Appendix 2. Secondly, to apply the integrated measure in planning we must reverse this calculation so as to calculate the required e.i.r.p. for each satellite so that all services achieve equal values of integrated quality. Details of this are derived in Appendix 3.

The evaluation of $I_{0.01}$ is summarised below in Section 4.1, with some observations about its behaviour. Section 4.2 sets out the steps in applying the equal-integrated-quality criterion.

4.1 The determination of $I_{0.01}$ and its behaviour as a function of system parameters

The equations which are used to determine the integrated quality for a given scenario are developed in Appendix 2. The expressions are easily evaluated by computer to produce a family of curves such as Fig. 6. This shows the integrated quality, $I_{0.01}$, as a function of X_{cs}, with separate curves for different values of A_1.

The range of A_1 shown in the figure covers most likely applications. The appropriate value of A_1 can in turn be obtained from curves such as Fig. 7(a), for 20 GHz, and Fig. 7(b), for 22.5 GHz. These are produced using the CCIR Rep. 564 rain model and calculating the elevation angle assuming that the satellite is located at a longitude which differs from that of the receiving location by 15 degrees. (Other similar curves could be produced for different values of relative longitude[3]). The 0.01%-year rainfall values can be obtained from the maps given in CCIR Rep. 563, of which one for Europe and Africa has been reproduced as Fig. 8. It can be seen that the

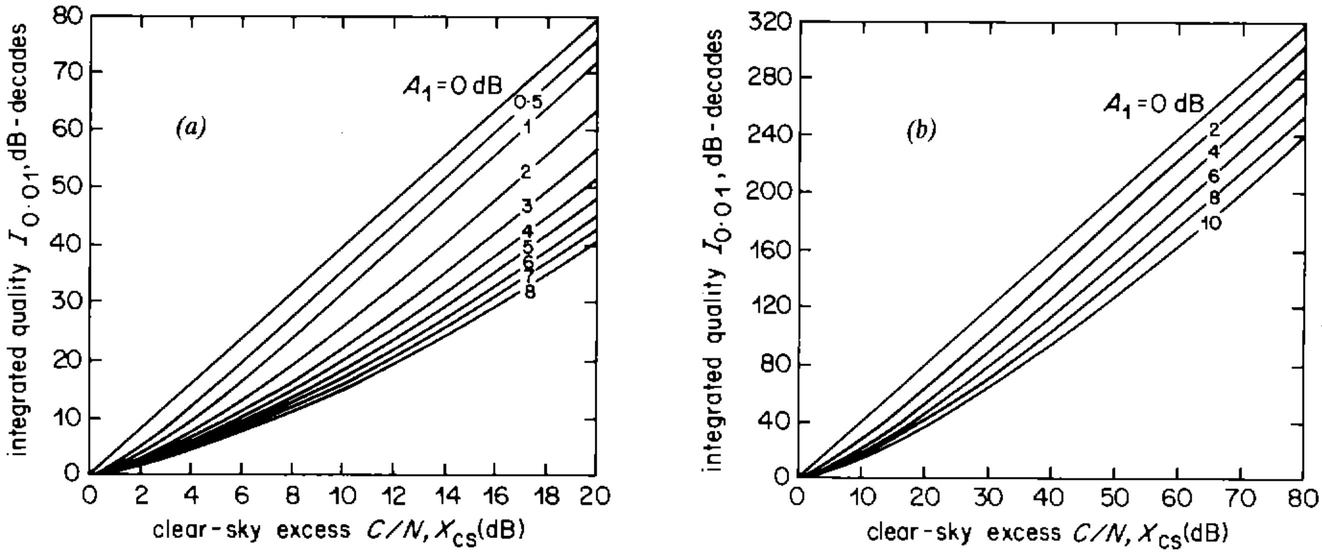

Fig. 6 - The dependence of integrated quality ($I_{0.01}$) on the clear-sky excess C/N (X_{cs}), for various values of 1%-time rain attenuation, (A_1). (a) and (b) show the variation at two different scales.

value of A_1 for the UK at 20 GHz is between 1 and 1.5 dB, while the highest value of A_1, in tropical Africa, is about 6.5 dB.

The effect of specifying that the same value of integrated quality $I_{0.01}$ should be achieved for all transmissions received in their respective service areas can be seen by drawing a horizontal line on Fig. 6. This will cut the various curves at values of X_{cs} ranging upwards from that for a hypothetical totally-dry climate, X_{dry}. The necessary range of X_{cs} to achieve equivalent $I_{0.01}$ increases as X_{dry} is increased from zero, until the point is reached where, for all values of A_1 of interest, the C/N does not go below threshold except for less than 0.01% year, i.e. below the lower limit of integration. For values of X_{dry} greater than this the curves are all parallel, and the required range of X_{cs} does not increase with further increase in X_{dry}. This is illustrated in Fig. 6(b), which is Fig. 6(a) plotted over a wider range of X.

4.2 Practical application of the equal-integrated-quality criterion

In practice what is needed is to set the e.i.r.p. of each satellite transmitter so that services to all countries achieve the same value of integrated quality. The necessary steps in this practical application of the criterion are thus as follows.

1. Select a value for the lower limit of integration time, say t_1 equal to 0.01%. The range of the

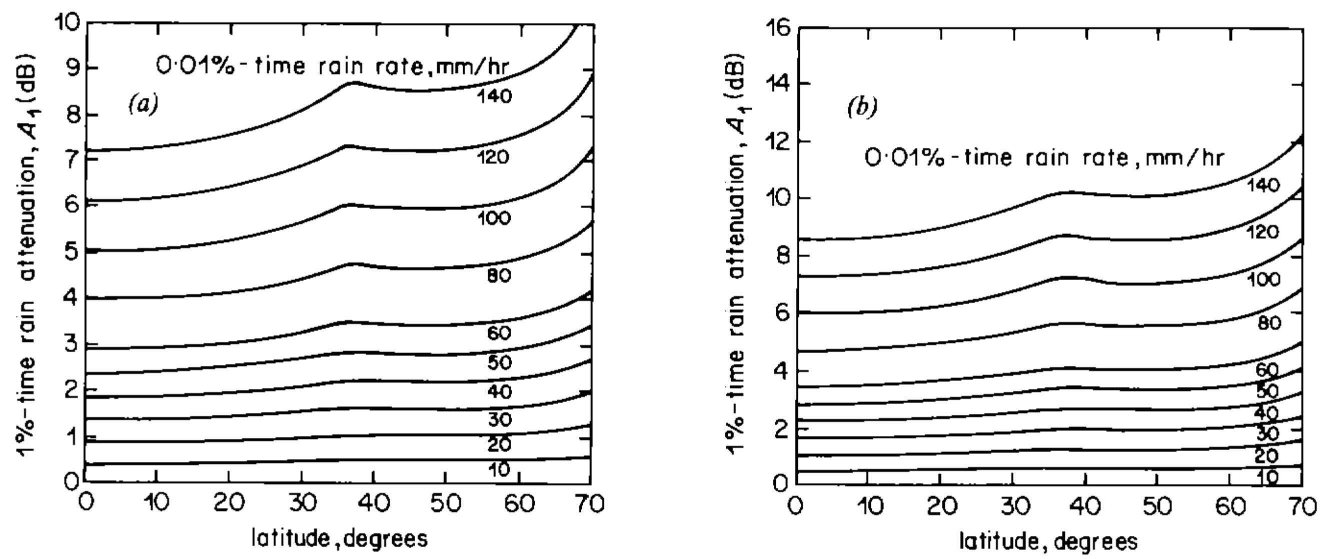

Fig. 7 - The variation of the slant-path rain attenuation exceeded for 1% time (A) as a function of receiver latitude for various 0.01%-time rain rates. The receiver location is assumed to be at sea level and to have a longitude of 15° relative to the satellite. (a) 20 GHz (b) 22.5 GHz.

integration is thus from 0.01% to 100% time, i.e. 4 decades, and the measure of integrated quality is written $I_{0.01}$.

2. Take as a reference an 'ideal' country having no rain. This would have a constant value of excess C/N, equal to X_{dry}, and its value of integrated quality $I_{0.01}$ would therefore be $4X_{dry}$ dB-decades.

3. Now choose a reference value of X_{dry} appropriate to the system being planned; a sensible example would be 10 dB. This might correspond to a C/N of 20 dB with a threshold value $C/N_{threshold}$ of 10 dB. The reference value of integrated quality $I_{0.01}$ is thus 40 dB-decades in this case.

4. All transmissions should now have their e.i.r.ps set so that with an agreed hypothetical receiver they would all achieve the same integrated quality, e.g $I_{0.01}$ = 40 dB-decades. This implies that the clear-sky value of excess C/N, X_{cs}, will, in all practical cases, exceed the 'dry' reference value X_{dry}, e.g. 10 dB. The appropriate value of X_{cs} is chosen for each transmission, according to the value A_1 of slant-path rain attenuation exceeded for 1% of the year, by computation as outlined in Appendix 3, or more readily, by using a graph such as Fig. 9 which summarises the results.

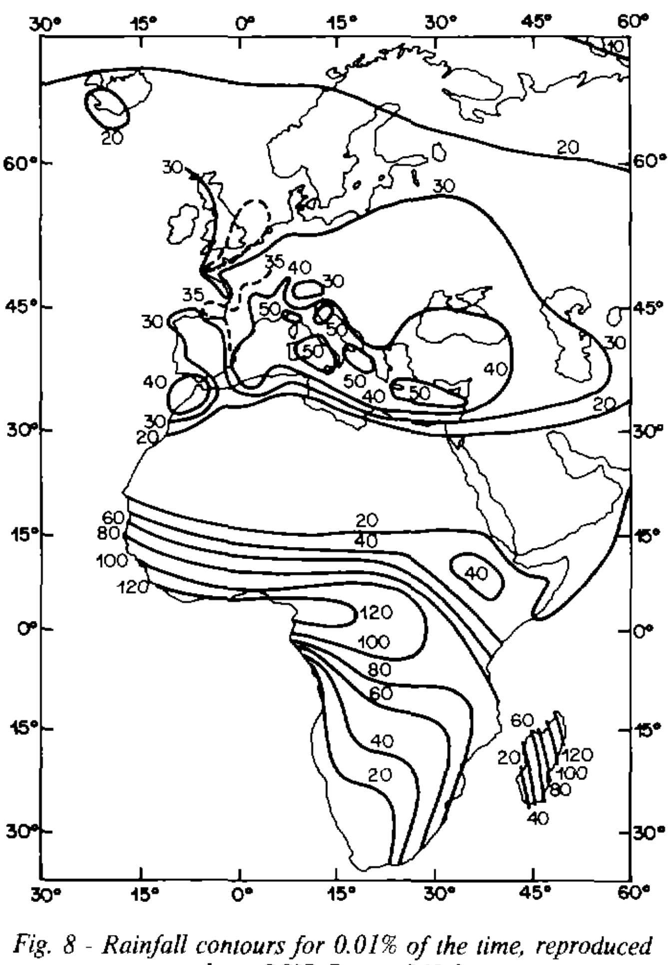

Fig. 8 - Rainfall contours for 0.01% of the time, reproduced from CCIR Report 563-3

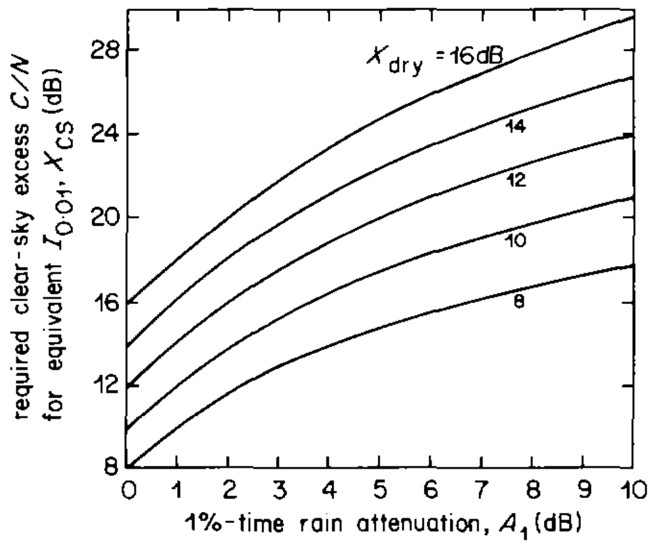

Fig. 9 - The dependence on the 1%-time rain attenuation (A_1) of the clear-sky excess C/N (X_{cs}) required to ensure equal integrated quality. X_{dry} is the constant excess C/N which would be enjoyed by a hypothetical country having a totally dry climate and which would give the same value of integrated quality.

5. COMPARISON OF INTEGRATED AND n%-WORST-MONTH CRITERIA

The n%-worst-month criterion was used in generating the 1977 WARC Plan. In that case the e.i.r.ps of all transmissions were initially adjusted so that all transmissions would give a C/N exceeding 14 dB for all but 1% of the worst month, for reception at the edge of the service area using an agreed hypothetical receiver. (A limit of 2 dB on the allowance for rain attenuation was then imposed, limiting the maximum e.i.r.p. for reasons of mutual interference, and thus departing from strict application of the criterion.)

It is of interest to compare this criterion with the equal-integrated-quality criterion proposed in this Report. We shall see that in certain circumstances they are equivalent.

Consider the case where the values of A_1 under consideration are sufficiently small that $X(t)$ is always greater than zero for t within the range of integration, t_1 to t_2.

From Appendix 2 we have, in general

$$I_{t_1} = X_{cs}\log_{10}(t_2/t_3) - A_1F(t_3), \qquad (5.1)$$

and, with the above-mentioned limitation on the behaviour of X,

$$I_{t_1} = X_{cs}\log_{10}(t_2/t_1) - A_1F(t_1), \qquad (5.2)$$

Substituting $k_1 = \log_{10}(t_2/t_1)$ and $k_2 = F(t_1)$, since they are both constant once the range of integration has been chosen, this becomes

$$I_{t_1} = k_1X_{cs} - k_2A_1. \qquad (5.3)$$

When we apply the concept of equal integrated quality we adjust the values of X_{cs} for each value of A_1 so that in every case I_{t_1} has the same value I_{ref}. If we postulate a reference example having $A_1 = 0$ and thus X is equal to X_{dry} for all t, we have I_{ref} is equal to k_1X_{dry}, so that for all other transmissions X_{cs} is related to A_1 by

$$I_{ref} = k_1X_{dry} = k_1X_{cs} - k_2A_1,$$

i.e. $$X_{cs} = (k_2/k_1)A_1 + X_{dry}. \qquad (5.4)$$

Now the value of X which is not exceeded for t % time is given by

$$X(t) = X_{cs} - A(t) = X_{cs} - A_1f(t)$$

(where $f(t)$ is the rain-model function defined in Appendix 2)

$$= X_{dry} + A_1[(k_2/k_1) - f(t)], \qquad (5.5)$$

so that for all curves

$$X(t') = X_{dry}$$

where t' is given by

$$f(t') = k_2/k_1 = F(t_1)/\log_{10}(t_2/t_1). \qquad (5.6)$$

Thus all curves of $X(t)$ versus t representing equal integrated quality will intersect at the same point, where $X = X_{dry}$ and $t = t'$, a function only of the choice of integration range, provided that the values of A_1 are small enough in relation to the target X_{dry}. This can be seen in Fig. 10(a), where the curves for $A_1 = 0$, 0.5, 1 and 2 dB all intersect at the same point, but those for greater values of A_1 do not. Fig. 10(b) is a magnified view of the relevant part of Fig. 10(a), showing the effect more clearly.

We conclude that, for small A_1, imposing the concept of equal integrated quality I_{t_1} produces the same result as imposing equal C/N not achieved for t'% time, where t_1 and t' are related as derived above. This relationship is plotted in Fig. 11, which shows that choosing $t_1 = 0.01$% time is equivalent to a t'%-year criterion, where $t' = 0.27$% year. This is very close to the values of 0.29% or 0.3% year which are variously taken to be equivalent to 1% *of the worst month.* The proposed $I_{0.01}$ criterion is thus virtually the same in effect as the previously-used 1%-worst-month criterion, (for small A_1) and can be considered as a natural extension of it.

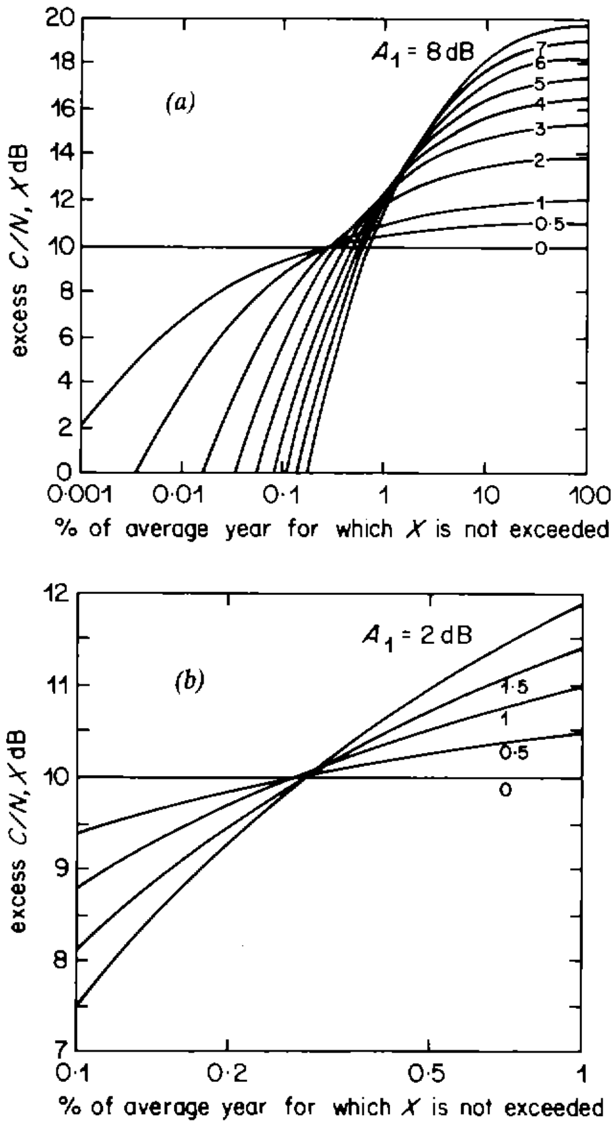

Fig. 10 - Curves of equal integrated quality, illustrating that they intersect at a single point for sufficiently small values of A_1, in this case below about 2 dB. (b) is a magnified version of the relevant portion of (a).

6. CONCLUSIONS

A new technique has been proposed for use in the international planning of direct-broadcast satellite services. It uses a newly-defined criterion, whereby an

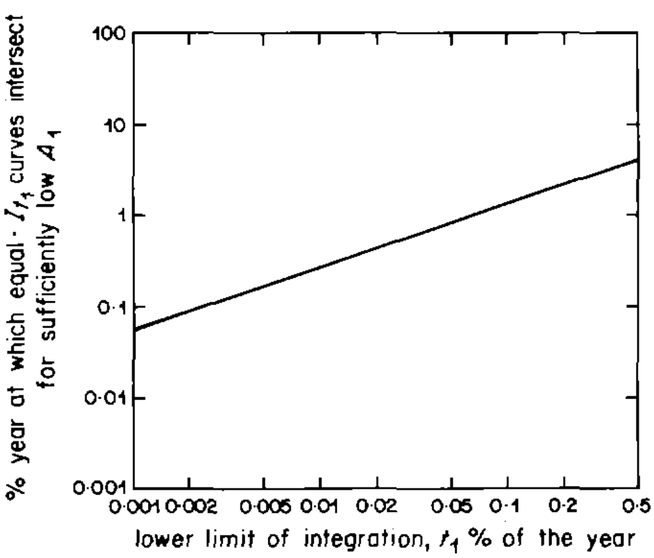

Fig. 11 - The dependence of t′, the % year at which curves of equal integrated quality I_{t_1} intersect (for small values of A_1), on the lower limit of integration, t_1. This illustrates the relationship between the equal-integrated-quality criterion and the n%-worst-month criterion.

'integrated quality' measure is made to have the same value for all circuits to provide for equitable planning. The technique is shown to be particularly useful for planning future services at higher frequencies where large ranges of rain attenuation occur.

The integrated quality is a measure of the mean quality, but weighted to compensate for the performance under poor propagation conditions. It is proposed, as the planning objective, to render the integrated quality equal for all circuits.

When the range of rain attenuation is small, the new technique gives results very similar to those achieved using the '1%-worst-month' criterion used at the 1977 World Satellite Broadcasting Conference. In general, the new technique is a natural extension of the old one, particularly suitable for use at high frequencies, whereby the need to use the crude device of a 'rain cap' is avoided.

When the range of rainfall attenuations is large, the new technique reduces the range of e.i.r.ps necessary to provide a service. This avoids the need for 'over-engineering' the satellite power requirements and permits equitable access to the geo-stationary orbit for all countries. The proposed new technique equalises the effective overall quality obtainable for services to all areas despite the range of rainfall characteristics.

It is recommended that this new technique should be commended to the CCIR, particularly with regard to its studies on HDTV satellite broadcasting, and that further study should be directed towards choosing appropriate quality values for particular future services.

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APPENDIX 1

Symbols

$A(t)$	the slant-path rain attenuation (in dB) equalled or exceeded for t % of the year
A_1	A(t) for t = 1% of the year
a, b	coefficients of the CCIR rain attenuation algorithm (see equation A2.6)
$c_{1,2,3}$	coefficients of an algorithm for rain attenuation for 1% to 100%. (See equation A2.8)
$D(t)$	degradation (in dB) of receiver's figure of merit by atmospheric attenuation equalled or exceeded for t % of the year
F	see equation A2.13
$f(t)$	interpolation function for rain-attenuation model (defined in equation A2.4)
g	arbitrary function
I	the integrated quality (defined in equation 3.1)
k_1, k_2	constants, see equation 5.14
K, L, M, N	see equation A2.17
$p(Y)$	probability distribution function of the variable Y
$\mathrm{P}(Y)$	cumulative distribution function of the variable Y
q	$\log_{10}(t)$
t	% time for which a given parameter is not achieved or exceeded
u	$\log_{10}\mathrm{e}$
$W(X)$	weighting function for excess C/N
X	the amount by which the achieved C/N exceeds a threshold C/N (defined in equation 3.2)
X_{cs}	the value of X in clear-sky conditions
X_{dry}	the value of X achieved by a hypothetical dry country
Y	arbitrary variable.

APPENDIX 2

Mathematical basis of calculation of $I_{0.01}$

We start from the definition of the integrated quality given in Section 3.2:

$$I_{t_1} = \int_{t=t_1}^{t=t_2} X \; d(\log_{10} t), \qquad \text{(A2.1) (same as 3.1)}$$

To evaluate this integral we must first express X as a function of t. To avoid repetition of the qualification "for (.) > 0", and so on, we introduce the notation $\{ g \}$, where:

$$\{ g \} = g, \quad g > 0, \qquad \text{(A2.2)}$$

$$= 0 \quad g \leqslant 0. \qquad \text{(Such brackets \{ \} are sometimes called Macaulay brackets).}$$

We can then write

$$X(t) = \{ X_{cs} - A(t) - D(t) \} \qquad \text{(A2.3)}$$

where

$X(t)$ is the excess C/N in dB, which is not achieved for t % of the average year;

X_{cs} is its clear-sky value, i.e. $X(100\%)$;

$A(t)$ is the slant-path rain attenuation, in dB, which is equalled or exceeded for t % of the average year; and

$D(t)$ is the degradation of the receiver figure-of-merit (G/T) which is equalled or exceeded for t % of the average year, and which is caused by the rainfall attenuation.

Since $D(t)$ is caused by the rain attenuation it could, for any particular receiver, be specified as a function of $A(t)$. Thus the total effect of rainfall, $A(t) + D(t)$, could in principle be expressed as some function $A'(t)$, say. However, for the present analysis it is proposed to neglect $D(t)$, for three reasons:

(i) it greatly simplifies the computation;

(ii) it makes the computation of the integrated quality independent of the choice of parameter values for the hypothetical receiver used in planning;

(iii) the resulting error is small, and its effect is not serious for planning purposes.

$A(t)$ can be taken from the current CCIR model for rainfall attenuation[6]. In this, the behaviour of $A(t)$ is taken to have the same shape for all countries and climates, but with a change of scale dependent on the local rainfall statistics and the slant-path geometry. Thus we may write

$$A(t) = A_1 f(t), \qquad \text{(A2.4)}$$

where

A_1 is the slant-path rain attenuation, in dB, which is equalled or exceeded for 1% of the average year, for the particular location and slant-path geometry; and

$f(t)$ is an interpolation function which is the same for all locations and paths.

Unfortunately, the CCIR model only gives a range for t in the range 0.001% to 1%, whereas we must perform the integration from t_1% to 100%. The lower limit presents no difficulty, since in practice we choose t_1 equal to 0.01%, but it is essential to define $f(t)$ for the range of t from 1% to 100%. Fortunately the attenuation, and thus also its range, for such time percentages is fairly small. (This is the very reason why little data of sufficient accuracy is available to enable the production of a model for this range of t). The precise choice of interpolation formula does not therefore have a very great bearing on the results. We know, from the CCIR model, the value and slope of $f(t)$ for $t = 1\%$, while for $t = 100\%$ we know (by definition) that $f(t) = 0$. It is also reasonable to set the slope equal to zero at $t = 100\%$. A simple cubic expression in log(t) is proposed, so that $f(t)$ may be written:

$$\begin{aligned} f(t) &= f_1(t), \quad 0.001\% \leq t \leq 1\%, \\ &= f_2(t), \quad 1\% < t \leq 100\%, \end{aligned} \tag{A2.5}$$

where:

$$f_1(t) = t^{-(a+bq)}, \tag{A2.6}$$

$$q = \log_{10} t, \tag{A2.7}$$

$$f_2(t) = 1 + c_1 q + c_2 q^2 + c_3 q^3, \tag{A2.8}$$

with

$$\left.\begin{aligned} a &= 0.546, \\ b &= 0.043, \end{aligned}\right\} \text{ from the CCIR model}^{6}$$

$$\left.\begin{aligned} c_1 &= -1.2572, \\ c_2 &= 0.5072, \\ c_3 &= -0.064306. \end{aligned}\right\} \text{ by curve-fitting}$$

These expressions can now be substituted into the evaluation of the integrated quality:

$$I_{t_1} = \int_{t=t_1}^{t=t_2} \{X_{cs} - A_1 f(t)\} \; d(\log_{10} t), \tag{A2.9}$$

A further minor substitution removes the inconvenience of the Macaulay brackets. Let t_0 be the value of t for which their contents become zero in the above expression. Clearly, t_0 is the percentage of time for which the C/N is below threshold, and it is given by

$$f(t_0) = X_{cs}/A_1. \tag{A2.10}$$

This can be solved for t_0. If $f(t_0)$ is greater than unity, then the solution reduces to a quadratic equation in ($\log_{10} t_0$), whereas if $f(t_0)$ is less than unity then a cubic equation in ($\log_{10} t_0$) must be solved.

The Macaulay brackets in the integral can then be replaced by conventional brackets provided the range of integration is from the greater of t_0 or t_1, i.e.:

$$I_{t_1} = \int_{t=t_3}^{t=t_2} (X_{cs} - A_1 f(t)) \; d(\log_{10} t), \tag{A2.11}$$

where

$$t_3 = t_1, \qquad \text{for } t_1 \geq t_0,$$

$$t_3 = t_0, \qquad \text{for } t_1 < t_0.$$

Remembering that X_{cs} and A_1 are constants for any given scenario, I_{t_1} becomes:

$$I_{t_1} = X_{cs} \log_{10}(t_2/t_3) - A_1 F(t_3), \tag{A2.12}$$

where

$$F(t_3) = \int_{t=t_3}^{t=t_2} f(t) \; d(\log_{10} t), \tag{A2.13}$$

$F(t_3)$ can be decomposed into two parts, assuming $t_3 < 1\%$ (which it probably should be for any acceptable system), so that:

$$F(t_3) = F_1(t_3) + F_2(1) \qquad \text{when } t_3 < 1\%, \tag{A2.14}$$

$$= F_2(t_3) \qquad \text{when } t_3 \geq 1\%,$$

where

$$F_1(t') = \int_{t=t'}^{t=1\%} f_1(t) \; d(\log_{10} t), \tag{A2.15}$$

and

$$F_2(t') = \int_{t=t'}^{t=t_2} f_2(t) \; d(\log_{10} t), \tag{A2.16}$$

It can be shown that $F_1(t') = N - K \operatorname{erf}(L \log_{10}(t') + M)$, (A2.17)

with

$$K = 152.3826168$$

$$L = 0.3146603868$$

$$M = 1.997727572$$

$$N = 151.6626226$$

and

$$F_2(t') = [q + c_1q^2/2 + c_2q^3/3 + c_3q^4/4]_{q=2} - [q + c_1q^2/2 + c_2q^3/3 + c_3q^4/4]_{q=\log t'} \tag{A2.18}$$

The equations presented here provide the basis for practical calculations.

APPENDIX 3

Calculation of e.i.r.ps to give equal integrated quality

Appendix 2 has shown how to calculate the integrated quality $I_{0.01}$ when the clear-sky excess C/N, X_{cs}, and 1%-time rain attenuation, A_1, are known, and Section 4.1 has also shown how the appropriate values of X_{cs} for a particular transmission to achieve a reference quality can be deduced from Fig. 6. In practice, once a reference quality has been decided on, say equivalent to X_{dry} = 10 dB, the only calculation which need be performed in planning is the latter problem of determining the required X_{cs} for a given A_1. This Appendix examines this further.

The value of clear-sky e.i.r.p. can be derived using simple numerical techniques.

Values for X_{cs} given A_1 and the desired value of $I_{0.01}$ are derived using a simple iterative procedure. There is no simple analytical procedure. This is for two reasons: first, the fact that different expressions are used for $f(t)$ and $F(t)$ for t above and below 1%; and secondly, the fact that reception may or may not go below threshold within the range of integration and so it is necessary to evaluate t_0 in each case. The desired value of X_{cs} is therefore obtained by numerical iteration. One possibility is based on a simplified Newton's method.

Newton's method provides a way to solve $g(x) = 0$ for x. Let the true root of the equation be x_0. An initial estimate for x_0 is made, say x_1. Writing $g'(x)$ for dg/dx,we may write the approximation

$$g(x_0) \;=\; 0 \simeq g(x_1) + (x_0 - x_1)g'(x_1), \qquad \text{(A3.1)}$$

i.e.

$$x_0 \simeq x_1 - g(x_1)/g'(x_1) \qquad \text{(A3.2)}$$

If we write $x_2 = x_1 - g(x_1)/g'(x_1)$, then x_2 is now a closer approximation to x_0 than x_2, so that x_2 can be substituted for x_1 and the process repeated until $g(x_1)$ is as close to 0 as desired.

In our case we want to make $I_{0.01}(X_{cs}, A_1)$ equal to some reference value I_{ref} by solving for X_{cs} with a given value of A_1. Thus, in our problem

$$g(X_{cs}) \;=\; I_{0.01}(X_{cs}, A_1) - I_{ref} \qquad \text{(A3.3)}$$

so that

$$X_{cs} \simeq X_{cs2} = X_{cs1} - [I_{0.01}(X_{cs1}, A_1) - I_{ref}]/(dI_{0.01}(X_{cs1}, A_1)/dX_{cs}). \qquad \text{(A3.4)}$$

Strict application of Newton's method thus requires the computation or estimation of $dI_{0.01}(X_{cs}, A_1)/dX_{cs}$ for $X_{cs}{=}X_{cs1}$. This could be done analytically (rather difficult in this case), or numerically, by taking the difference between values of $I_{0.01}$ computed for $X_{cs}{=}X_{cs1}$ and $X_{cs}{=}X_{cs1} + \delta$, where δ is a small increment. However, a simpler proposal is to note that for the hypothetical 'dry' country the slope $dI_{0.01}/dX_{cs} = 4$. The slope for all other curves is some value between 0 and 4. If we substitute the value 4 in the iteration formula it becomes

$$X_{cs2} \;=\; X_{cs1} - [I_{0.01}(X_{cs1}, A_1) - I_{ref}]/4. \qquad \text{(A3.5)}$$

If we take as the initial estimate $X_{cs1}{=}X_{dry}$, and perform the iteration given above, the process still converges despite the simplification and the desired value of X_{cs} can easily be computed for any value of A_1.

The iteration formula described above was used to produce the curves in Fig. 9 from which values of X_{cs} to give equal integrated quality can be read off directly as a function of A_1. The values of X_{cs} so derived can then be used to plot curves of X versus t which satisfy the equal-integrated-quality criterion, as shown in Fig. 5.

APPENDIX 4

The mathematical significance of the criterion

In this Appendix we show how the equal-integrated-quality criterion is related to some other planning methods.

A4.1 Revision of Statistics - p.d.fs, c.d.fs and means

This sub-section briefly revises a few statistical concepts, chiefly in order to standardise notation for the analysis of the next sub-section.

Consider a real variable Y which varies in some way with time. It is often useful, particularly when the variation is random rather than deterministic, to have some way to describe or quantify the variation.

The most well-known statistical measure is the *mean*. The mean value $\overline{Y}$ of a variable Y is defined over some time interval t_1 to t_2 by

$$Y = \frac{1}{(t_2-t_1)} \int_{t_1}^{t_2} Y(t)\ dt. \tag{A4.1}$$

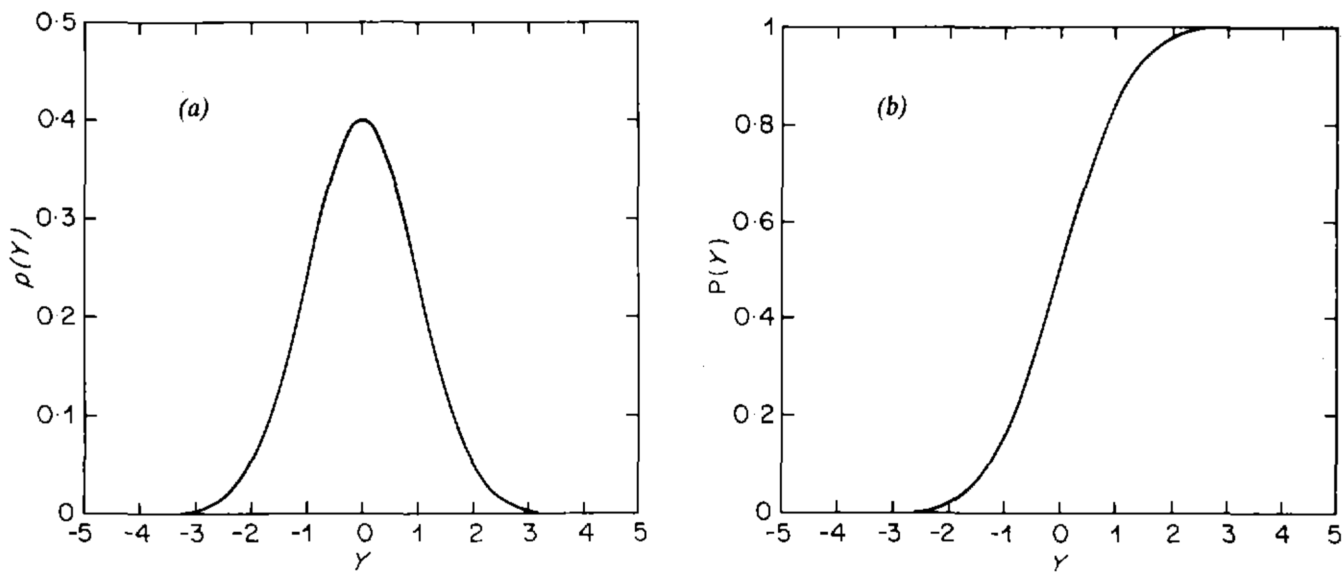

Fig. A4.1 - Example of distribution functions:
(a) a probability distribution function (p.d.f.). (b) its cumulative distribution function (c.d.f.)

This parameter $\overline{Y}$ gives us the average value of Y, but it does not tell us anything about its variation. One way to characterise the latter is to plot a *probability distribution function (p.d.f.)*, $p(Y)$ against Y. The p.d.f., $p(Y)$, is defined so that $p(Y)\mathrm{d}Y$ is the probability that Y lies in the infinitesimal range Y to $Y{=}dY$. The probability that Y lies in some finite range is then given by the integral

$$\Pr(Y_1 < Y < Y_2) = \int_{Y_1}^{Y_2} p(Y)\, dY. \tag{A4.2}$$

An obvious constraint on $p(Y)$ results because Y must always take some value between $\pm\infty$, so that

$$\int_{-\infty}^{+\infty} p(Y)\ dY = 1. \tag{A4.3}$$

Another function of Y can be derived from the p.d.f. Define $P(Y)$ as the probability that the value of the variable does not exceed Y. $P(Y)$ is called the *cumulative distribution function*, and is clearly given by

$$P(Y) = \int_{-\infty}^{Y} p(Y)\, dY. \tag{A4.4}$$

The c.d.f. $P(Y)$ can thus be interpreted as the area under the curve of the p.d.f. $p(Y)$, from minus infinity to Y. Equivalently, the p.d.f. $p(Y)$ can be derived as the slope of the c.d.f., $p(Y) = dP(Y)/dY$. This is illustrated in Figs. A.4 (a) and (b).

The mean value of Y can be derived from the p.d.f.:

$$\overline{Y} = \int_{-\infty}^{+\infty} Y p(Y)\, dY. \tag{A4.5}$$

A4.2 The criterion explained as a weighted mean quality

In the definition of integrated quality we make use of the function $X(t)$. It is in effect a kind of inverse c.d.f. for the excess C/N, because it gives the value X of the excess C/N which is not exceeded for t% of the year. Clearly, $t/100$ is the c.d.f. $P(X)$, i.e. the probability that the value X is not exceeded. Let us now translate the integrated quality measure into these terms.

Recall the definition of the integrated quality measure, I_{t_1}:

$$I_{t_1} = \int_{t=t_1}^{t=100\%} X\, d(\log_{10} t). \qquad \text{(A4.6) (as 3.1)}$$

We have seen that $t = 100\, P(X)$, so that

$$\begin{aligned} \log_{10} t &= 2 + \log_{10} P(X), \text{ and thus} \\ d(\log_{10} t) &= d(\log_{10} P(X)) \\ &= \frac{u\, dP}{P(X)}, \end{aligned} \tag{A4.7}$$

where $u = \log_{10} e$.

Furthermore, since $p(X) = dP(X)/dX$, we can write $dP = p(X)\, dX$. Making these substitutions, and writing X_1 for the value of $X(t_1)$ we obtain

$$I_{t_1} = \int_{X_1}^{X_{cs}} \frac{u}{P(X)}\, X p(X)\, dX. \tag{A4.8}$$

Compare this result with

$$\overline{X} = \int_{\text{all } X} X\, p(X)\, dX. \qquad \text{(A4.9) (as A4.5)}$$

In effect we are calculating a weighted mean of X,

$$I_{t_1} = X_{\text{wtd mean}} = \int_{\text{all } X} W(X)\, X p(X)\, dX, \tag{A4.10}$$

where the weighting function $W(X)$ is given by

$$\begin{aligned} W(X) &= 0 \qquad X < X_1, \\ &= \frac{u}{P(X)} \qquad X > X_1 \qquad \text{(i.e. within the defined integration range for } I_{t_1}\text{).} \end{aligned} \tag{A4.11}$$

The integrated quality measure I_{t_1} is thus effectively a weighted-mean excess C/N, where the weighting has two distinct properties:

(i) no account whatsoever is taken of the performance during the worst t_1% of the year, regardless of whether or not the C/N is above threshold for some of this time;

(ii) for the remaining $(100\text{-}t_1)$% of the year, more importance is attached to a small increase in quality at the low-quality, small %-time end of the scale than to the same small increase applied to the highest quality.

The weighting thus serves precisely those purposes which were outlined in Section 3, when the need for a new criterion was identified. It is of interest to note that if the integrated measure were to be calculated as the area under a curve of $X(t)$ versus t using a linear scale for t (instead of the logarithmic scale proposed), and covering the whole range of t from 0 to 100%, then the result would be a true, unweighted mean value of X. The choice of a logarithmic scale for t is therefore important; it is the means whereby the subjectively-important property (ii) above has been achieved in the proposed criterion.

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